

Core Mathematics C2 Paper E

1. A sequence of terms is defined by

$$u_n = 3^n - 2, \quad n \geq 1.$$

- (i) Write down the first four terms of the sequence. [1]

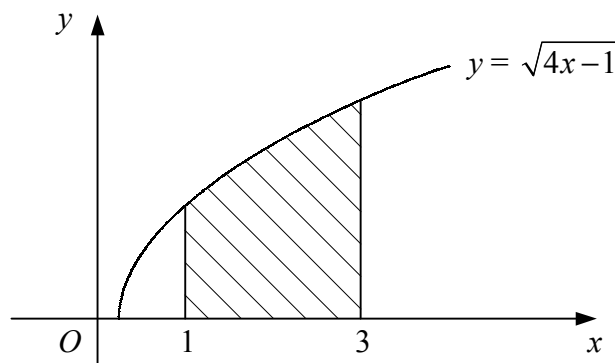
The same sequence can also be defined by the recurrence relation

$$u_{n+1} = au_n + b, \quad n \geq 1, \quad u_1 = 1,$$

where a and b are constants.

- (ii) Find the values of a and b . [4]

- 2.



The diagram shows the curve with equation $y = \sqrt{4x-1}$.

- (i) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region bounded by the curve, the x -axis and the lines $x = 1$ and $x = 3$. [4]
- (ii) Explain briefly how you could use the trapezium rule to obtain a more accurate estimate of the area of the shaded region. [1]
3. (i) Expand $(2 + y)^6$ in ascending powers of y as far as the term in y^3 , simplifying each coefficient. [4]
- (ii) Hence expand $(2 + x - x^2)^6$ in ascending powers of x as far as the term in x^3 , simplifying each coefficient. [3]

4.
$$f(x) = \frac{4}{2 + \sin x^\circ}.$$

(i) State the maximum value of $f(x)$ and the smallest positive value of x for which $f(x)$ takes this value. [2]

(ii) Solve the equation $f(x) = 3$ for $0 \leq x \leq 360$, giving your answers to 1 decimal place. [5]

5. (a) Given that $t = \log_3 x$,

(i) write down an expression in terms of t for $\log_3 x^2$, [1]

(ii) show that $\log_9 x = \frac{1}{2}t$. [4]

(b) Hence, or otherwise, find to 3 significant figures the value of x such that

$$\log_3 x^2 - \log_9 x = 4. \quad [3]$$

6. Given that

$$\frac{dy}{dx} = \frac{x^3 - 4}{x^3}, \quad x \neq 0,$$

and that $y = 0$ when $x = -1$, find the value of y when $x = 2$. [8]

7. A student completes a mathematics course and begins to work through past exam papers. He completes the first paper in 2 hours and the second in 1 hour 54 minutes.

Assuming that the times he takes to complete successive papers form a geometric sequence,

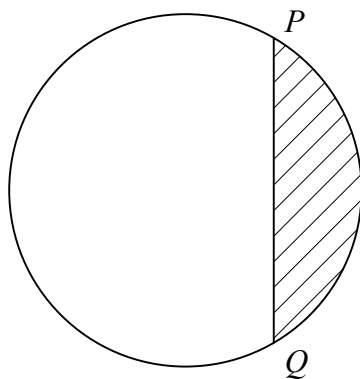
(i) find, to the nearest minute, how long he will take to complete the fifth paper, [3]

(ii) show that the total time he takes to complete the first eight papers is approximately 13 hours 28 minutes, [2]

(iii) find the least number of papers he must work through if he is to complete a paper in less than one hour. [4]

Turn over

8.



The diagram shows a circle of radius 12 cm which passes through the points P and Q . The chord PQ subtends an angle of 120° at the centre of the circle.

- (i) Find the exact length of the major arc PQ . [2]
- (ii) Show that the perimeter of the shaded minor segment is given by $k(2\pi + 3\sqrt{3})$ cm, where k is an integer to be found. [4]
- (iii) Find, to 1 decimal place, the area of the shaded minor segment as a percentage of the area of the circle. [4]

9.

$$f(x) = x^3 - 9x^2 + 24x - 16.$$

- (i) Evaluate $f(1)$ and hence state a linear factor of $f(x)$. [2]
- (ii) Show that $f(x)$ can be expressed in the form $f(x) = (x + p)(x + q)^2$, where p and q are integers to be found. [4]
- (iii) Sketch the curve $y = f(x)$. [2]
- (iv) Using integration, find the area of the region enclosed by the curve $y = f(x)$ and the x -axis. [5]